CHAPTER THREE

Binomials and factorization

Multiplication of binomials:

a) $(a+b)(c+d) = a \times c + a \times d + b \times c + b \times d = ac + ad + bc + bd$

Example (1)

$(2+x)(4+b) = 2 \times 4 + 2 \times b + x \times 4 + x \times b = 8 + 2b + 4x + xb.$

Example (2)

$(2x+1)(x+3) = 2x \times x + 2x \times 3 + 1 \times x + 1 \times 3 = 2x^2 + 6x + x + 3$
$= 2x^2 + 7x + 3.$

Example (3)

$(x^2+5)(x+2) = x^2 \times x + x^2 \times 2 + 5 \times x + 5 \times 2 = x^3 + 2x^2 + 5x + 10.$

b) $(a+b)(c-d) = a \times c - a \times d + b \times c - b \times d = ac - ad + bc - bd$

Example (1)

$(2+x)(4-y) = 2 \times 4 - 2 \times y + x \times 4 - x \times y = 8 - 2y + 4x - xy.$

Example (2)

$(3x+2)(4-x) = 3x \times 4 - 3x \times x + 2 \times 4 - 2 \times x$

c) $(a-b)(c+d) = a \times c + a \times d - b \times c - b \times d = ac + ad - bc - bd$

Example (1)

$(2x-1)(a+3) = 2x \times a + 2x \times 3 - 1 \times a - 1 \times 3 = 2ax + 6x - a - 3.$

Example (2) $(y-2)(2y+3) = y \times 2y + y \times 3 - 2 \times 2y - 2 \times 3 = 2y^2 + 3y - 4y - 6.$

Example (3) $(x^2-2)(b+y) = x^2 \times b + x^2 \times y - 2 \times b - 2 \times y = x^2b + x^2y - 2b - 2y.$

d) $(a-b)(c-d) = a \times c - a \times d - b \times c + b \times d = ac - ad - bc + bd.$

Example (1) $(x-2)(2x-4) = x \times 2x - x \times 4 - 2 \times 2x + 2 \times 4 = 2x^2 - 4x - 4x + 8 = 2x^2 - 8x + 8.$

Example (2) $(3x-4)(x-2) = 3x \times x - 3x \times 2 - 4 \times x + 4 \times 2 = 3x^2 - 6x - 4x + 8 = 3x^2 - 10x + 8.$

Q1. Multiply (3*x*+y) by (*x*+2y).

Soln

$$(3x+y)(x+2y) = 3x \times x + 3x \times 2y + y \times x + y \times 2y$$
$$= 3x^2 + 6xy + yx + 2y^2 = 3x^2 + 7xy + 2y^2$$

$Q2.$ Multiply $(2x+3)by(4x-5)$

Soln.

$$(2x+3)(4x-5) = 2x \times 4x - 2x \times 5 + 3 \times 4x - 3 \times 5$$
$$= 8x^2 - 10x + 12x - 15 = 8x^2 + 2x - 15.$$

Q3. Expand (4a – 3b) (c + 6d)

Soln.

(4a – 3b) (c + 6d) $= 4a \times c + 4a \times 6d - 3b \times c - 3b \times 6d = 4ac + 24ad - 3bc - 18bd$

Q4. Expand $(2x-5)(3x-2)$

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Soln

$(2x-5)(3x-2) = 2x \times 3x - 2x \times 2 - 5 \times 3x + 5 \times 2$
$= 6x^2 - 4x - 15x + 10 = 6x^2 - 19x + 10.$

Q5. Expand $(2x + y)^2$

Soln.

$(2x + y)^2 = (2x + y)(2x + y) = 2x \times 2x + 2x \times y + y \times 2x + y \times y = 4x^2 + 2xy + 2xy + y^2 = 4x^2 + 4xy + y^2$

Q6. Expand $(3a - 2b)^2$

Soln.

$(3a - 2b)^2 = (3a - 2b)(3a - 2b) = 3a \times 3a - 3a \times 2b - 2b \times 3a + 2b \times 2b = 9a^2 - 6ab - 6ba + 4b^2 = 9a^2 - 12ab + 4b^2$

Q7. Expand and simplify the following:

a) $(2a + 2b)(a - b) + (2a - b)(a - b)$

Soln.

Considering the first part, $(2a + 2b)(a - b) = 2a \times a - 2a \times b + 2b \times a - 2b \times b = 2a^2 - 2ab + 2ba - 2b^2 = 2a^2 - 2ab + 2ab - 2b^2 = 2a^2 - 2b^2$

Considering the second part, $(2a - b)(a - b) = 2a \times a - 2a \times b - b \times a + b^2 = 2a^2 - 2ab - ba + b^2 = 2a^2 - 3ab + b^2$ $Therefore$ $(2a + 2b)(a - b) + (2a - b)(a - b) = (2a^2 - 2b^2) + (2a^2 - 3ab + b^2) = 2a^2 + 2a^2 - 2b^2 + b^2 - 3ab = 4a^2 - b^2 - 3ab.$

b. $(4x + 2y)(2x - 3y) - (x - y)(3x + 2y)$

Soln.

Considering the first part, $(4x + 2y)(2x - 3y) = 4x \times 2x - 4x \times 3y + 2y \times 2x - 2y \times 3y = 8x^2 - 12xy + 4yx - 6y^2 = 8x^2 - 12xy + 4xy - 6y^2 = 8x^2 - 8xy - 6y^2$

Considering the second part, $(x - y)(3x + 2y) = x \times 3x + x \times 2y - y \times 3x - y \times 2y = 3x^2 + 2xy - 3xy - 2y^2 = 3x^2 - xy - 2y^2$ $Therefore$

$(4x+2y)(2x-3y)-(x-y)(3x+2y)$

$= (8x^2-8xy-6y^2)-(3x^2-xy-2y^2)$

$= 8x^2-8xy-6y^2-3x^2+xy+2y^2$

$= 8x^2-3x^2-8xy+xy-6y^2+2y^2 = 5x^2-7xy-4y^2$

b) $(2p+q)(3u-2v)-(p-2q)(u-v)$

. Soln

$$(2p+q)(3u-2v) = 2p\times 3u - 2p\times 2v + 2q\times 3u$$

$-q\times 2v = 6pu-4pv+3qu-2qv.$

$Also\ (p-2q)(u-v) = p\times u - p\times v - 2q\times u + q\times v$

$= pu-pv-2qu+2qv$

Therefore $(2p+q)(3u-2v)-(p-2q)(u-v)$

$= (6pu-4pv+3qu-2qv)-(pu-pv-2qu+2qv)$

$= 6pu-4pv+3qu-2qv-pu+pv+2qu-2qv$

$= 6pu-pu-4pv+pv+3qu+2qu-2qv-2qv$

$= 5pu-3pv+5qu-4qv$

d) $(2x-y)^2+(x-y)(3x-2y)$

Soln.

$(2x-y)^2 = (2x-y)(2x-y) = 2x\times 2x - 2x\times y - y\times 2x + y\times y$

$= 4x^2-2xy-2xy+y^2 = 4x^2-4xy+y^2$

$(x-y)(3x-2y) = x\times 3x - x\times 2y - y\times 3x + y\times 2y$

$= 3x^2-2xy-3xy+2y^2 = 3x^2-5xy+2y^2$

$\therefore (2x-y)^2+(x-y)(3x-2y)$

$= 4x^2-4xy+y^2+3x^2-5xy+2y^2 = 7x^2-9xy+3y^2$

e) $(2r-t)^2-(3r+t)^2$

$Soln.$

$(2r-t)^2 = (2r-t)(2r-t)$

$= 2r \times 2r - 2r \times t - t \times 2r + t \times t = 4r^2 - 2rt - 2rt + t^2$

$= 4r^2 - 4rt + t^2$

$(3r + t)^2 = (3r + t)(3r + t) = 3r \times 3r + 3r \times t + t \times 3r + t \times t$

$= 9r^2 + 3rt + 3tr + t^2 = 9r^2 + 3rt + 3rt + t^2$

$= 9r^2 + 6rt + t^2$

$\therefore (2r - t)^2 - (3r + t)^2 = (4r^2 - 4rt + t^2) - (9r^2 + 6rt + t^2)$

$= 4r^2 - 4rt + t^2 - 9r^2 - 6rt - t^2$

$= 4r^2 - 9r^2 - 4rt - 6rt + t^2 - t^2 = -5r^2 - 10rt + 0$

$= -5r^2 - 10rt$

$Q8$. Simplify the following:

a. $(2a + 3b)(3a + 5b)$

Soln.

$(2a + 3b)(3a + 5b) = 2a \times 3a + 2a \times 5b + 3b \times 3a + 3b \times 5b$

$= 6a^2 + 10ab + 9ba + 15b^2 = 6a^2 + 10ab + 9ab + 15b^2$

$= 6a^2 + 19ab + 15b^2$

b. $\frac{5}{x+y} + \frac{2}{2y-1}$

Soln.

$$\frac{5}{x+y} + \frac{2}{2y-1} = \frac{5(2y-1) + 2(x+y)}{(x+y)(2y-1)} = \frac{10y - 5 + 2x + 2y}{(x+y)(2y-1)}$$

$$= \frac{10y + 2y + 2x - 5}{(x+y)(2y-1)} = \frac{12y + 2x - 5}{(x+y)(2y-1)}$$